

# Depinning and wetting in nonequilibrium systems

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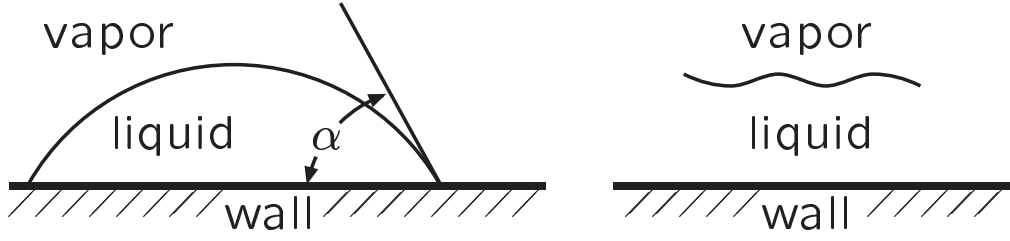
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**Abstract.** We present an extension of equilibrium wetting to nonequilibrium situations particularly suited to systems with anisotropic interactions. Both critical and complete wetting transitions were found and characterized. We have identified a region in the space of parameters (temperature and chemical potential) where the wet and non-wet phases coexist. Emphasis is made on the analogies and differences between equilibrium and nonequilibrium wetting.

## EQUILIBRIUM WETTING

Imagine that a small amount of liquid is poured on a substrate. At two-phase equilibrium, i.e. a static situation where the liquid is at equilibrium with its vapor, it may happen that the liquid does not coat the substrate, in which case it beads as droplets characterized by a contact angle  $\alpha$  as shown in figure 1. This is called partial wetting and the contact angle is related to the surface tensions,  $\sigma$ , of the intervening interfaces through Young's formula (dating back to 1805)  $\sigma_{sv} = \sigma_{sl} + \sigma_{lv} \cos \alpha$ , where  $\sigma_{sv}$  is the substrate/vapor surface tension and so on. If, by contrast, the liquid spreads over the substrate and coats it uniformly (zero contact angle), the substrate is said to be wet by the liquid. A wetting transition occurs when, by changing the temperature, the substrate changes from a partially wet to wet state.

It is instructive to study the same phenomenon from a different point of view. Figure 2 depicts the phase diagram of a pure substance. It is clear from the above discussion that if wetting is to occur, the system has to be at liquid/vapor coexistence, with an arbitrary fraction in the liquid with the remainder in the vapor phase. The effect due to the presence of a substrate that adsorbs preferentially the liquid is also displayed in Figure 2: above a certain temperature,  $T_w$ , called the wetting temperature, the substrate is wet, while for  $T < T_w$  the substrate is not wet at coexistence. This is illustrated on the right of the same figure where the thickness of the wetting layer,  $h$ , is displayed as a function of the temperature and the chemical potential difference between the liquid and vapor phases,  $\mu$ , along three different paths: (1) the substrate remains not wet when coexistence is reached; (2) the thickness of the wetting layer diverges continuously as coexistence is approached from the gas phase (this is termed *complete wetting*); (3) as  $T$  approaches



**FIGURE 1.** Left, liquid droplet at equilibrium with its vapor on a (non-wet) planar substrate (wall); right, same situation for a uniform coating (wetting) of the substrate.

$T_w$  at coexistence the thickness of the wetting layer may either diverge continuously (denoted *critical wetting*) or discontinuously at  $T_w$  (denoted *first – order wetting*).

Assuming the system free energy to be a functional,  $\mathcal{H}$ , solely of the height  $h(\mathbf{x})$  of the liquid/vapor interface above the substrate coordinate  $\mathbf{x}$ , then [1]

$$\mathcal{H} = \int dx \left[ \frac{\sigma}{2} (\nabla h)^2 + V(h(x)) - \mu h \right] \quad (1)$$

where  $\sigma$  is the surface tension and  $V(h)$  accounts for the effective potential between the substrate and the interface. If all the interactions are short-ranged, then it can be proved that for large  $h$   $V$  has the form  $V(h) = b(T)e^{-h/\xi} + e^{-2h/\xi}$ , with  $b(T)$  proportional to  $T_w$  and  $\xi$  being the bulk correlation length [1].

One obtains a dynamic model of wetting by relating the time derivative of  $h$  with (minus) the functional derivative of  $\mathcal{H}$  through,

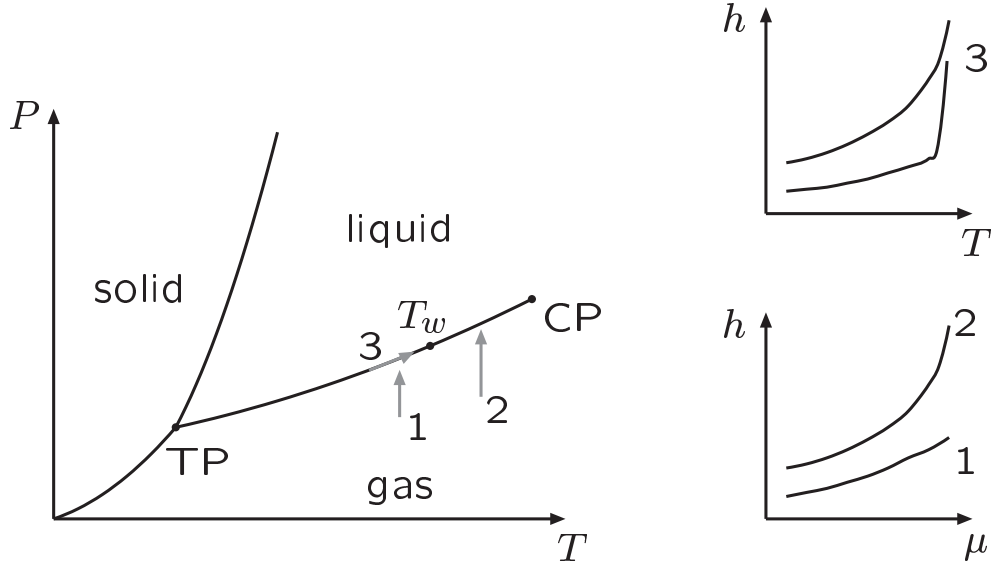
$$\frac{\partial h(\mathbf{x}, t)}{\partial t} = -\frac{\delta \mathcal{H}}{\delta h} + \eta(\mathbf{x}, t) = \sigma \nabla^2 h - \frac{\partial V(h)}{\partial h} + \mu + \eta, \quad (2)$$

where  $\eta$  is a Gaussian with noise with mean  $\langle \eta(\mathbf{x}, t) \rangle = 0$  and correlations  $\langle \eta(\mathbf{x}, t) \eta(\mathbf{x}', t') \rangle = 2D \delta(\mathbf{x} - \mathbf{x}') \delta(t - t')$ . In this context wetting appears in the guise of an unbinding transition and its phenomenology is described by the long-time behavior of the solutions of (2) as follows: (i) liquid/vapor coexistence obtains at  $\mu = \mu_c = 0$ , since for this value of the chemical potential difference the free interface does not move on average irrespective of its initial position. This is no longer the case when the system is at contact with a substrate. Under these circumstances there is a value  $b_w$ , (proportional to) the temperature, above which  $\langle h \rangle \rightarrow \infty$  as  $t \rightarrow \infty$  at coexistence,  $\mu = 0$  (critical wetting). Complete wetting corresponds to the divergence of  $\langle h \rangle$  as  $t \rightarrow \infty$  and  $\mu \rightarrow 0^-$  for values of  $b > b_w$  [2].

## NONEQUILIBRIUM WETTING

Consider the following Langevin equation,

$$\frac{\partial h(\mathbf{x}, t)}{\partial t} = \sigma \nabla^2 h + \lambda (\nabla h)^2 - \frac{\partial V(h)}{\partial h} + \eta \quad (3)$$



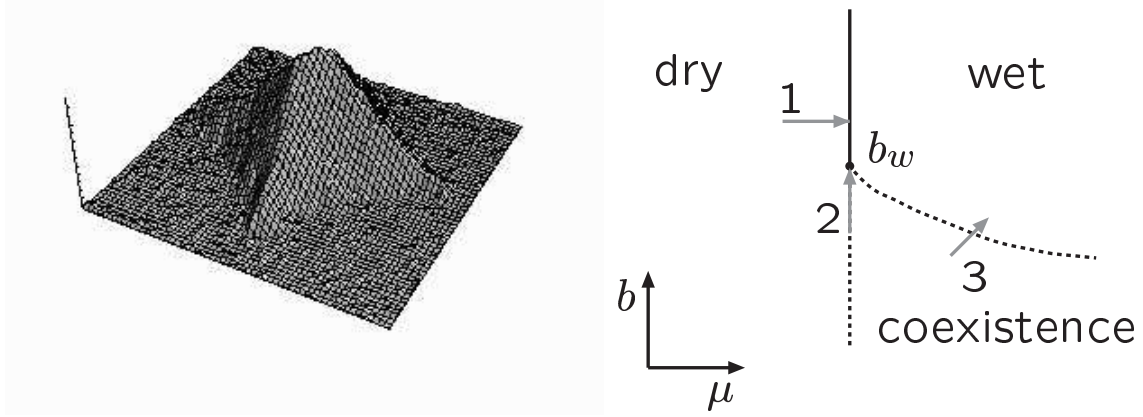
**FIGURE 2.** Left, pressure vs. temperature phase diagram of a pure substance.  $TP$ ,  $CP$  and  $T_w$  stand for the triple point, critical point and wetting temperature, respectively; right, thickness of the wetting layer as a function of the temperature and chemical potential difference for the paths indicated on the left.

which differs from (2) by the presence a new non-linearity, namely, the  $KPZ$  term  $\lambda(\nabla h)^2$ . In this section we will study the wetting properties of a system described by (3). The absence of an equilibrium Hamiltonian for (3) shows that it is a genuine non-equilibrium equation and justifies the title of this section.

There are a number of good reasons to include the  $KPZ$  term. It may be interpreted as a force acting on the tilted parts of the interface in the direction of growth and, therefore, it is relevant in systems with anisotropic interactions, where the growth of tilted interfaces may depend on their orientation. In fact, it governs the growth of crystals from atomic beams when desorption is allowed [3]. Further, a renormalization group study has shown that it is always generated, except when excluded by symmetry, when elastic objects depin in the presence of anisotropy [4]. Finally, lattice models of nonequilibrium wetting seem to be controlled by the  $KPZ$  non-linearity [5].

Investigating the wetting behavior of (3) requires carrying out the steps outlined in the previous section, a programme we have completed for  $\lambda = -\sigma = -1$ . We would like to stress that while the value of the surface tension is irrelevant, the sign of  $\lambda$  determines the behavior of (3). By contrast with the equilibrium system, bulk coexistence no longer obtains at  $\mu = 0$ . Rather,  $\mu_c = \langle (\nabla h)^2 \rangle$  that for one-dimensional substrates is given approximately by  $\mu_c = -D\lambda/(2\sigma\Lambda)$ , where  $\Lambda$  is a lattice cutoff [6]. In higher dimensionalities one has to resort to numerical methods to obtain  $\mu_c$ .

Our findings are sketched in figure 3. Nonequilibrium critical wetting occurs as  $b \rightarrow b_w = -0.32 \pm 0.05$  along path 1.  $\langle h \rangle$  diverges as  $|b - b_w|^\beta$ , with  $\beta = -2.6 \pm 0.2$ . This value differs from that of equilibrium wetting,  $\beta = -1$  [1] thereby defining a new universality class as expected. Along path 2 nonequilibrium complete wetting is observed as  $\mu \rightarrow \mu_c$  with  $b > b_w$ . The associated exponent in this case is  $-0.41$ , with



**FIGURE 3.** Right, phase diagram from eq. (3); left, typical structures observed in the coexistence region for a two-dimensional substrate.

error bars that exclude the equilibrium value  $\beta = -1/3$ .

As we have said before, no wetting transition can occur below the wetting temperature. Pushing a little bit further (within this interfacial model) one would expect a depinning transition when crossing the boundary  $\mu = \mu_c$ , for  $\mu > \mu_c$  is the realm of the liquid phase. Interestingly enough, it turns out that the vapor phase is stable up to  $\mu = \mu^*(b) > \mu_c$  (path 3 of figure 3). In fact, within the region delimited by the dashed lines in figure 3, the dipinned and pinned phases coexist. As in equilibrium, this means that the system will either exhibit a vapor (pinned) or liquid (depinned) phase depending on the initial conditions. The fact that the coexistence region is finite rather than a line is, however, a nonequilibrium effect. The microscopic mechanism underlying this behavior is illustrated on the left of figure 3: when a bound interface makes a large excursion away from the substrate, marking (in principle) the onset of the depinning transition, the fluctuation acquires a triangular shape (pyramidal for two-dimensional substrates), that is pushed down due to the KPZ term and, eventually, suppressed. This mechanism operates in a finite region of the space of parameters  $(T, \mu)$  and explains why the coexistence region is finite [5, 7, 8].

## CONCLUSIONS

We have proposed and solved a continuum model for nonequilibrium wetting. It consists of a dynamic version of a well known equilibrium wetting Hamiltonian supplemented with a KPZ non-linear term. We focussed on short-ranged forces and negative nonlinearities. We found and characterized in detail the nonequilibrium counterparts of critical and complete wetting transitions. A finite region of coexistence of wet and nonwet phases, the existence of triangular (pyramidal) patterns within this region and the relation to depinning transitions below the wetting temperature have also been discussed.

## ACKNOWLEDGMENTS

We acknowledge financial support from the E.U. through Contract No. ERBFM-RXCT980183, by the Ministerio de Ciencia y Tecnología (FEDER) under project BFM2001-2841 and from the Fundação para a Ciência e a Tecnologia, contract SFRH/BPD/5654/2001.

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